Reliability of Unidirectional Fibrous Composites

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The paper deals with a fundamental method to evaluate the reliability of undirectional fibrous composites under any plane stress condition, and the effects of various factors on the reliability are investigated. It is found that the proposed method based on a recent structural reliability theory is useful and that the variations of principal strengths, applied stresses, and orientation angles reduce the reliability. The emphasis is placed on the orientation angle along which the maximum reliability is obtained, and it is found that the optimum angle varies with the variation of the applied stress in some cases.

Nomenclature

$CV(X)$	= coefficient of variation of X
$E(X)$	= mean value of X
F_{ij},F_i	= coefficients in failure criterion
M	= safety margin
P_f	= failure probability
R_s	= longitudinal shear strength
R_x,R_y	= tensile strength
$R_{x'},R_{y'}$	= compressive strength
S_i,Si	= stress
U,u	= transformed basic variables in standard normal space
x	= axis along fibers
y	= axis perpendicular to fiber direction
β	= safety index
μ_M	= mean of safety margin
θ	= fiber orientation angle
σ_M	= standard deviation of safety margin

Subscripts

s	= shear in x-y axes
x	= longitudinal
y	= transversal
1	= major reference axis
2	= minor reference axis, perpendicular to 1 axis
6	= shear in 1 to 2 axes

Introduction

STUDIES on the reliability of the static strength of fibrous composites can be classified into three groups: 1) to investigate experimentally the factors that affect the variation or the scatter of the strength using a number of specimens,[1,2] 2) to analyze the variation of the strength theoretically using micromechanical models,[3-7] and 3) to analyze the reliability of the strength of unidirectional and laminated composites using a macroscopic failure criterion and fundamental data on the variations of the strengths along the principal directions.[8,9] Among these studies, there are many studies on items 1 and 2, and the factors affecting the variation of the strength have become clear to some extent. But the studies on item 3 are very few.

The strength and stiffness of composite materials change remarkably by changing the kinds, volume contents, and orientations of their reinforcing fibers and stacking sequences. Therefore, optimum material design can be performed under a given loading condition. From this standpoint, optimum material design methods have been developed under criteria on maximum bending stiffness,[10] maximum in-plane strength,[11-13] maximum bending strength,[14] maximum buckling strength,[15] and maximum fundamental frequency.[16] It is found from these studies that a maximum performance can be obtained with an optimum laminate configuration, but sometimes the optimum fiber orientations change discontinuously when the loading condition changes and the performances have large sensitivities to the design variables and loading conditions. Therefore, the best configurations studied so far can be optimum under deterministic conditions, but many problems remain to be solved under probabilistic conditions.

A method for evaluating the reliability of composites under probabilistic conditions is proposed in this paper. This study is the first step to establish an optimum material design method based on reliability.

Reliability Analysis

Failure Criterion for Composites

There are some criteria proposed for the failure of unidirectional fibrous composites with respect to their principal axes: 1) maximum stress, 2) maximum strain, 3) Hill[17] theory, 4) Hoffman[18] theory, and 5) Tsai-Wu[19] theory. Among those, the Tsai-Wu criterion has been used by many researchers and it is recognized as the most general criterion for undirectional composites. Tsai-Wu criterion has the form

$$F_{xx}S_x^2 + 2F_{xy}S_xS_y + F_{yy}S_y^2 + F_{ss}S_s^2 + F_xS_x + F_yS_y = 1 \quad (1)$$

where

$$F_{xx} = 1/R_xR'_x, \qquad F_x = 1/R_x - 1/R'_x$$
$$F_{yy} = 1/R_yR'_y, \qquad F_y = 1/R_y - 1/R'_y$$
$$F_{ss} = 1/R_s^2, \qquad F_{xy} = F^*_{xy}\sqrt{F_{xx}F_{yy}}$$

and subscripts x and y denote the fiber and its perpendicular directions, respectively, and subscript s denotes shear, and the prime denotes compressive strength. Factor F^*_{xy} is assumed to be $-1/2$.

In this paper, the Tsai-Wu criterion is used, but any criterion can be used to evaluate the reliability by the proposed method.

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Table 1 Materials constants used,[20] MPa

Material	Type	R_x	R'_x	R_y	R'_y	R_s
T300/5208	Graphite/epoxy	1500	1500	40	246	68

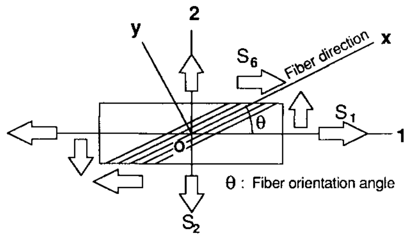


Fig. 1 Coordinate systems for unidirectional composites.

For failure under any plane stress condition, off-axis (not along the material principal axes) stresses are transformed into on-axis (along the principal axes) stresses to use on-axis failure criteria. The coordinate systems are shown in Fig. 1, where 1 and 2 represent reference axes and θ is the angle between 1 axis and x axis. The typical material constants of the composite used for calculations are shown in Table 1.[20]

Advanced First-Order Second-Moment Method

A mathematical expression for the failure of unidirectional composites is assumed to be given as follows:

$$M(x) = g(X_1, X_2, \ldots, X_n) \le 0 \tag{2}$$

where M is safety margin and X means n basic variables that affect the strength. $M \le 0$ means failure and $M > 0$ means nonfailure. The failure probability P_f can be calculated by using the joint probability density function $f_x(X_1, X_2, \ldots, X_n)$:

$$P_f = \iint \cdots \int_D f_x(X_1, X_2, \ldots, X_n)\, \mathrm{d}X_1 \cdot \mathrm{d}X_2 \ldots \mathrm{d}X_n \tag{3}$$

where D is the region where $M \le 0$.

A thin unidirectional composite plate under in-plane loading condition is considered here, as shown in Fig. 1. Applied stresses S_1, S_2, and S_6 then become basic variables instead of applied loads. Other basic variables are strengths along the fiber direction R_x, R'_x, R_y, R'_y, and R_s and fiber orientation angle θ. If multidirectional laminates are considered, the applied loads and each orientation of the laminated plies should be taken as basic variables.

The function g is an appropriate failure criterion. Here, the Tsai-Wu criterion, which is most popular for unidirectional lamina, is used. The safety margin M then becomes as follows:

$$M = 1 - \left\{ \sum_{i=x}^{y} \left[\left(\frac{1}{R_i} - \frac{1}{R'_i} \right) S_i + \frac{S_i^2}{R_i R'_i} \right] + \frac{S_s^2}{R_s^2} + \frac{2F^*_{xy} S_x S_y}{\sqrt{R_x R'_x R_y R'_y}} \right\} \tag{4}$$

where factor F^*_{xy} is assumed to be $-1/2$.

The number of basic random variables is eight when the fiber orientation angle is assumed to be deterministic. The integration of Eq. (3) in such a high-dimensional space is difficult, and function $f_x(X)$ is not given when sufficient statistical data are not obtained. Consequently, a reliability analysis is carried out by using the first and second moments (mean and variance) of the safety margin M and a safety index β.

When safety margin M is linear and the basic variables are normally distributed, safety index β is defined as follows[21]:

$$\beta = \mu_M / \sigma_M \tag{5}$$

where μ_M and σ_M are the mean and the standard deviation of the safety margin M, respectively. When the safety margin is nonlinear, the approximate values of μ_M and σ_M are obtained by linearizing the safety margin, that is, the safety margin is expressed by the Taylor series and the linear term is retained.

For the linearization of the safety margin, the linearization point where the Taylor series is evaluated affects the mean and the variance of the safety margin. If the mean valued point of the basic variables is adopted as the linearization point, then safety index β can be obtained analytically, but the safety index seems to be overestimated. This method is called the first-order second-moment (FOSM) method.

The definition for the safety index, Eq. (5), introduces difficulty in evaluating the safety index because it varies according to mathematical forms of the functions of the safety margin, even if the functions are equivalent to each other. Consequently, a new safety index that is independent of failure functions has been proposed by Hasofer and Lind.[22] In the case of independent basic variables, basic variables X_i are transformed into standard normal variables U_i as follows:

$$U_i = \Phi^{-1}[F_{x_i}(X_i)] \tag{6}$$

where $F_{x_i}(X_i)$ and Φ are the probability distribution function of X_i and standard normal distribution function, respectively. Then the safety margin in X space is transformed to the safety margin in U space:

$$h(\boldsymbol{u}) = M(X) \tag{7}$$

The linearization of the limit state function is carried out at such a point $\boldsymbol{u}^*$ that yields the shortest distance between the point on the failure surface $h(\boldsymbol{u}) = 0$ and the origin in U space. Point $\boldsymbol{u}^*$ is called a β point, and safety index β is given as the distance between the origin and the point $\boldsymbol{u}^*$:

$$\beta = \min(\boldsymbol{u}^{*T} \cdot \boldsymbol{u}^*)^{1/2} \tag{8}$$

A β point usually is obtained by using an iterative method[22-24] or a nonlinear mathematical programming method.[25] This method is called the advanced first-order second-moment (AFOSM) method. The superiority of the AFOSM method over the FOSM method is well known.[26]

The search for a β point is reduced to the following constrained optimization problem:

$$\text{Minimize } f = (\boldsymbol{u}^T \cdot \boldsymbol{u})^{1/2} \qquad \text{subject to } h(\boldsymbol{u}) = 0 \tag{9}$$

Proposed Method

To solve this problem efficiently, the following method has been proposed.[27] An extended Lagrangian function is introduced as follows:

$$L_r(\boldsymbol{u}, \mu) = (\boldsymbol{u}^T \cdot \boldsymbol{u})^{1/2} + \mu |h(\boldsymbol{u})| + 0.5\, r[h(\boldsymbol{u})]^2 \tag{10}$$

where μ and r are constants ($\mu, r > 0$).

Equation (10) can be solved easily by making use of an unconstrained optimization technique. An algorithmic procedure is as follows.

Step 1: Specify the initial values of μ and r (e.g., $r^0 = 5$, $\mu^0 = 0$) and set $k = 0$, $r^k = r^0$, and $\mu^k = \mu^0$.

Step 2: Input the initial value of $\boldsymbol{u}$, i.e., $\boldsymbol{u}^k = \boldsymbol{u}^0$ (e.g., $u_i^0 = 0$, $i = 1, 2, \ldots, n$).

Step 3: Solve the unconstrained optimization problem of minimizing $L_r(\boldsymbol{u}, \mu)$ by a conjugate gradient method, then, obtaining solution $\boldsymbol{u}^{k+1}$.

Step 4: If the convergence condition $|h(\boldsymbol{u}^{k+1})| < \varepsilon$ is satisfied for a sufficiently small value of $\varepsilon (>0)$ stop the calculation, otherwise, go to step 5.

Step 5: Set $r^{k+1} = \omega \cdot r^k$, $\mu^{k+1} = \mu^k + r^k \cdot h(\boldsymbol{u}^k)$, where ω is a constant ($\omega > 0$) (e.g., $\omega = 5$).

Step 6: Set $k = k + 1$, then go to step 3.

By substituting the optimum solution $\boldsymbol{u}^*$ thus obtained, into Eq. (8), the probability of failure is estimated as

$$P_f = \Phi(-\beta) \tag{11}$$

β-Point Searching Method for Multimodal Limit State Function

When a limit state function is not unimodal, that is, there exists more than one local minima, $\boldsymbol{u}^*$ is dependent on the initial value $\boldsymbol{u}^0$. Consequently, it is not sure that a global minimum is obtained. The limit state function discussed here is multimodal, therefore, an extension is performed.

Consider that point $\boldsymbol{u}^*$ is rotated around the origin through arbitrary angle. If point $\boldsymbol{u}^*$ has a global minimum length to the limit state function, the loci of the point always exist in the safety region after any rotational movement. Otherwise, that is, when point $\boldsymbol{u}^*$ is a local minimum, the terminus of the rotated point may exist in the failure region after a rotation as seen from a multimodal limit state function of Fig. 2. Then another local minimum point is searched that has a shorter length between the point and the origin than the previous point. For a multimodal limit state function, β point can be obtained by repeating the procedure.

The algorithm is as follows:

Step 1: Input $\boldsymbol{u}^0$, i.e., the initial value of $\boldsymbol{u}$.

Step 2: Obtain a local minimum point $\boldsymbol{u}^*$ by using the extended Lagrangian function.

Step 3: Set $i = 0$, $\boldsymbol{u}^i = \boldsymbol{u}^*$.

Step 4: Generate uniform random numbers, determine the rotation angle from the random numbers and rotate $\boldsymbol{u}^i$ to $\boldsymbol{u}^{i+1}$, i.e., $\boldsymbol{u}^{i+1} = T\boldsymbol{u}^i$.

Step 5: If $h(\boldsymbol{u}^{i+1}) > 0$ then go to step 6, otherwise go to step 7.

Step 6: Set $i = i + 1$. If $i \geq$ limit then set $\boldsymbol{u}^* = \boldsymbol{u}^i$ and terminate the calculation, otherwise go to step 4.

Step 7: Find α such that $h(\alpha \boldsymbol{u}^{i+1}) = 0$, and set $\boldsymbol{u}^0 = \alpha \boldsymbol{u}^{i+1}$, and go to step 2.

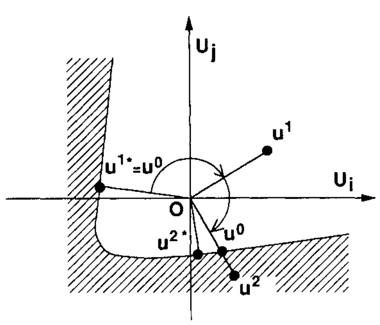


Fig. 2 Method for finding the global minimum for multimodal limit state function.

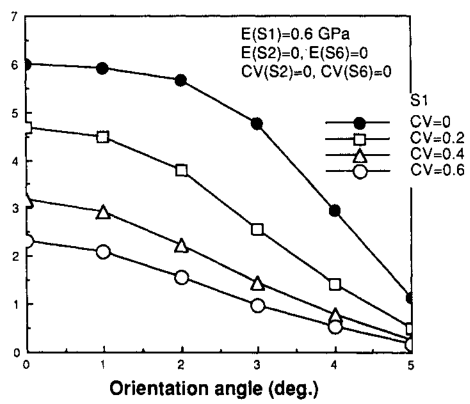


Fig. 3 Effect of the coefficient of variation of the 1-axis tensile stress under uniaxial loading condition.

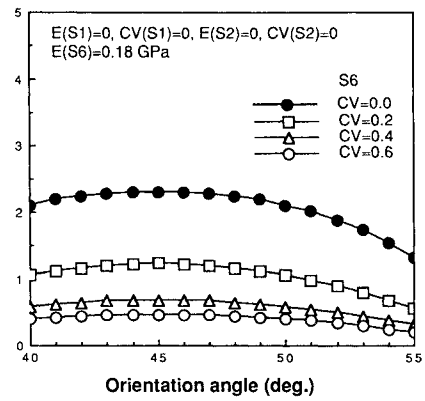


Fig. 4 Effect of the coefficient of variation of 1-axis shear stress under uniaxial loading condition.

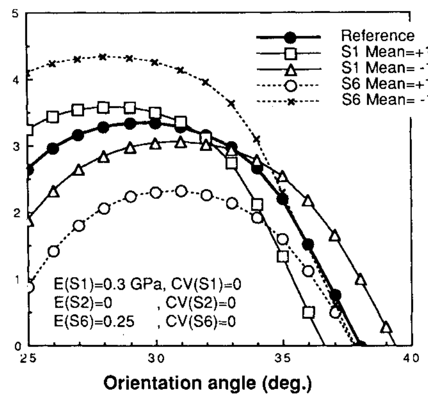


Fig. 5 Effects of the mean values of S_1 and S_6.

Results and Discussion

Strength Parameters Used for Calculations

The material used for the calculations is a typical Graphite/Epoxy (T300/5208) and their strengths are assumed to be normally distributed. The mean values of the strengths along the fiber direction are shown in Table 1. The coefficients of variation are all assumed to be 0.1, for simplicity.

The applied stresses S_1, S_2, and S_6 are also assumed to be normally distributed, and the respective mean values and the coefficients of variation are given appropriately. The coefficient of variation of the applied stress is varied with loading conditions. If the possibility of the change in the load is large and its mean value is small, then the coefficient becomes very large. The values from 0 to 60% are used for the coefficient of variation of the applied stress in this paper.

It is assumed that the strength and the applied stresses have no correlation with each other, except for the last section where the effect of the correlation is discussed.

Reliability Under Uniaxial Stress

Effect of Mean Value of Stress

The effects of the mean values of the uniaxial tensile, compressive, and shear stresses on the reliability of composites are apparent; that is, the safety index decreases with an increase in the mean value of the applied stress. The fiber orientation angle that yields the maximum reliability is 0 deg for uniaxial tension and compression and 45 deg for pure shear.

Effect of Variation of Stress

Figure 3 shows the effect of the coefficient of variation (CV) of the 1-axis tensile stress and Fig. 4 shows the effect of

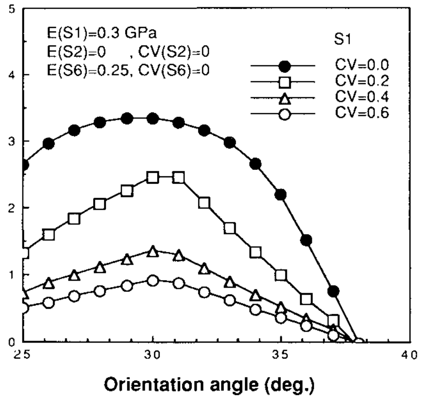


Fig. 6 Effect of the coefficients of variation of S_1.

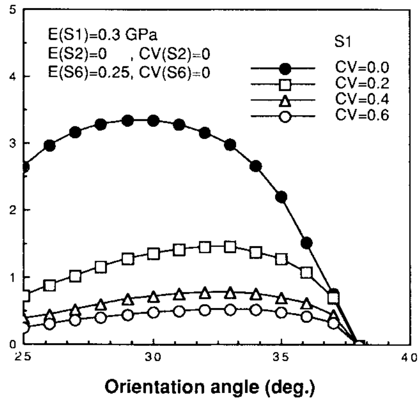


Fig. 7 Effect of the coefficient of variation of S_6.

the CV of the shear stress under pure shear loading where the mean values of the respective stresses are constant. It is easily recognized that the reliability decreases when the CV increases. For the optimum design of composite structures, the orientation angle that yields a maximum reliability becomes very important. Such an orientation angle in this paper is referred to as the orientation of maximum reliability. From Figs. 3 and 4, the orientations of maximum reliability are found to be unaffected by the mean values or the CVs of the 1-axis tensile and the shear stresses under uniaxial loading condition.

Reliability Under Plane Stress

Effect of Mean Value of Stress

Figure 5 shows the relation between the safety index β and the orientation angle θ under 1-axis tensile and shear stresses. The values of stresses S_1 and S_6 were selected as 0.3 and 0.25 GPa, respectively, since this combination was found to be appropriate to clarify each effect of the applied stresses. The effect of S_6 reduces when S_1 increases, and vice versa. The heavy solid line shows the reference stress condition and the maximum reliability is obtained at about $\theta = 30$ deg. The light solid lines show the results for the mean value of 1-axis tensile stress μ_{S1} being increased and decreased by 10%, respectively. The safety index increases at small angle ($\theta < 32$ deg) and it decreases as μ_{S1} increases at a large angle ($\theta > 32$ deg). The orientation of maximum reliability increases with a decrease in μ_{S1}.

The broken lines show the results for the mean value of the shear stress μ_{S6} being changed by 10%. The safety index decreases as μ_{S6} increases and the orientation of maximum reliability decreases with a decrease in μ_{S6}.

The results are summarized as follows.

1) There exists an angle that gives the maximum reliability under any stress condition.

2) The angle that gives a maximum reliability approaches zero as the ratio of the tensile stress to the shear stress increases. On the other hand, it approaches 45 deg as the ratio decreases.

3) The reliability increases even when the stress increases at a certain orientation angle. In contrast with this, at another orientation angle the opposite phenomenon is observed, i.e., the reliability decreases when the stress decreases.

Item 3 is very important for optimum material designs. Figure 5 shows that the reliability can be increased by increasing the tensile stress at $\theta = 30$ deg, for example. The direction as well as the magnitude of the principal stress will cause a critical result.

Effect of Variation of Stress

Figure 6 shows the relation between safety index β and orientation angle θ under 1-axis tensile and shear stresses, and the effect of the CV of the tensile stress is shown. The safety index decreases as the CV increases and the orientation of maximum reliability changes a little. Figure 7 shows the effect of the CV of the shear stress. The safety index decreases as the CV increases and the orientation of maximum reliability changes considerably.

Comparison Between AFOSM and FOSM Methods

For nonlinear limit state functions, the value of the safety index depends on the linearization point of the function. The FOSM method adopts a mean value point as the linearization point, whereas the AFOSM method adopts a β point as described previously. The advantage of the FOSM method is the easiness of calculation since it gives an analytical expression but it sometimes overestimates reliability. The superiority of the AFOSM method over the FOSM method is well known.[26]

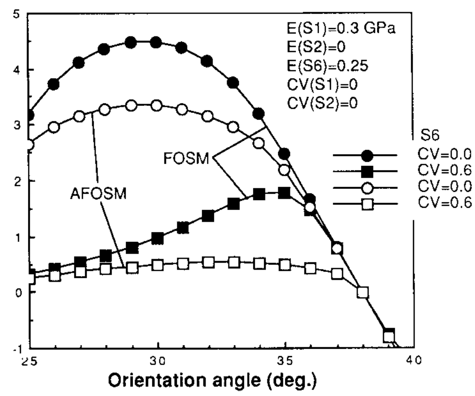


Fig. 8 Comparison between AFOSM and FOSM methods on the effect of CV of S_6.

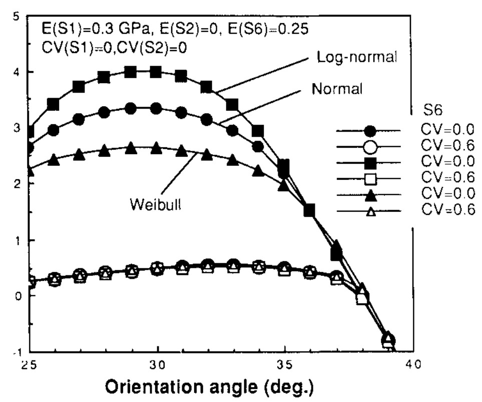


Fig. 9 Difference due to strength distributions (the effect of the CV of S_6 also is shown).

The results shown up to here are all obtained by the AFOSM method, but the following is the comparison between the AFOSM and FOSM methods.

Figure 8 shows the effect of the CV of S_6 on the safety index. From the figure, the following observations are taken.

1) The value of β is overestimated where $\beta > 0$, and this tendency becomes remarkable with increasing β.

2) The FOSM method sometimes introduces erroneous results about the orientation of maximum reliability.

Effect of Strength Distributions

Normal distributions are assumed for the strengths and the load variables up to here. Such a treatment is considered reasonable in carrying out a reliability analysis based on statistical data of composite materials. However, there exist negative values for strength variables when they are normally distributed, and a problem occurs in searching a β point by using Eq. (4) because it contains a square root. Therefore, the computations have been performed here by imposing an additional constraint that the strength variables are positive.

The probability distributions that do not take negative values and are often used as strength distributions are the log normal and Weibull distributions. Figure 9 shows the difference among various strength distributions. The results are summarized as follows.

1) Compared with the normal distribution, the log-normal distribution gives larger safety index and the Weibull distribution gives smaller. This tendency becomes remarkable when β is greater than about 2.

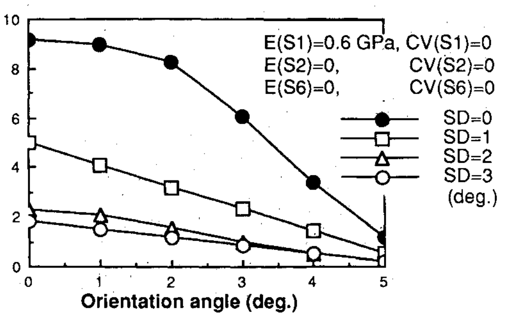
2) The fiber orientation angles that give maximum reliabilities are almost the same among those distributions.

3) There is no difference when the variation of the stress is large.

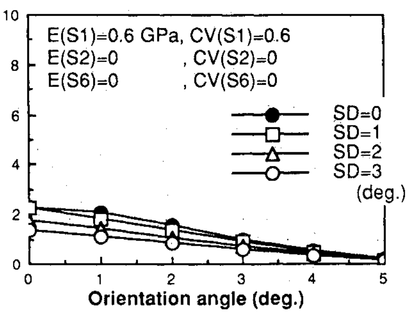
Effect of Variations of Orientation Angle

Although the orientation angle is treated as a deterministic variable in other sections, it is treated as a random variable here, and the effect of its variation is considered.

The effects of the variations of the orientation angle under uniaxial tensile loading condition are shown in Figs. 10,



a) With no variation of the applied stress



b) With a variation of the applied stress

Fig. 10 Effect of the standard deviation of the orientation angle under 1-axis tensile condition.

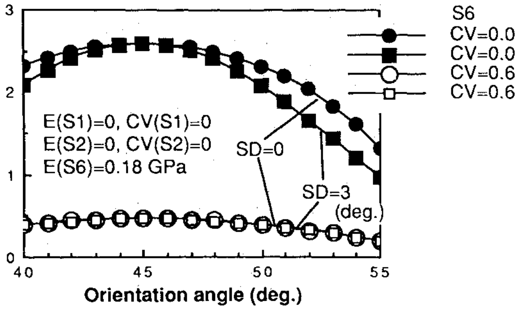


Fig. 11 Effect of the standard deviation of the orientation angle under 1-axis shear stress condition.

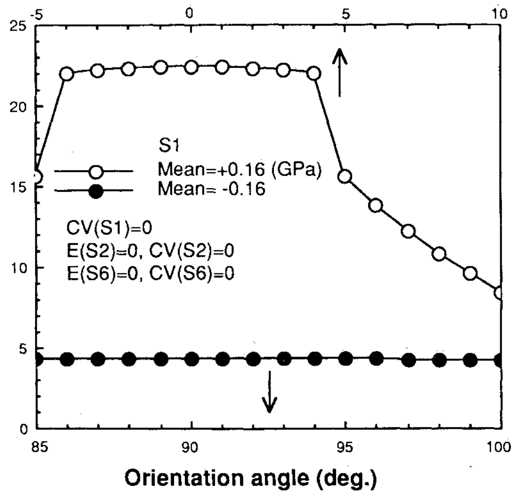


Fig. 12 Effect of the orientation angle under the plane stress condition of 1-axis tensile and 2-axis compression with equal magnitudes.

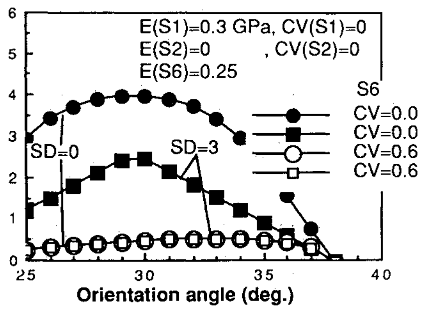
where Fig. 10a shows the case for the CV of the applied stress being zero, and Fig. 10b shows the case for the CV being 0.6.

It is found that the variation of the orientation angle reduces the reliability, and the reduction of the reliability decreases as the variation of the orientation angle increases. Therefore, the accuracy of the orientations in the fabrication process becomes less strict for applied stresses with large randomness.

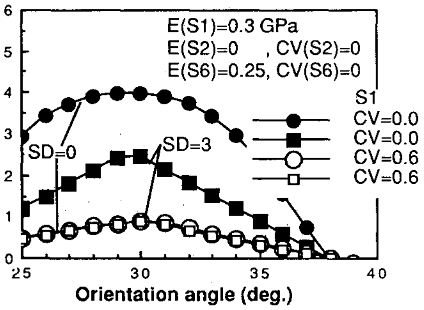
The effect of the variation of the orientation angle on the orientation angle under uniaxial shear stress condition is shown in Fig. 11. It is found that the maximum reliability is not affected by the variation of the orientation angle for both cases where the applied stress has a variation or not. This means that the accuracy of the orientation in fabrication is not required for this case as long as the orientation angles are normally distributed.

The insensitivity of the reliability to the variations of orientations is considered. The pure shear stress condition can be considered as the sum of the tensile and compressive stresses in 45-deg directions with equal magnitude, respectively. Under these tensile and compressive stresses, the respective safety index is obtained as in Fig. 12. It is clear that the failure is governed by the compressive stress, and the sensitivity of the safety index to the angle is very small.

For a general plane stress condition, the effect of the variation of the orientation angle appears as shown in Figs. 13 where the mean values of the 1-axis tensile and shear stresses are 0.3 and 0.25 GPa, respectively. Figure 13a shows the case with the variation of the shear stress. It is found that the reliability decreases remarkably with the increase in the variation of the orientation angles when the variation of the applied stress is small. However, the reliability does not change with the variation of the orientation angle when the applied stress has large randomness.



a) Case with the variation of the tensile stress



b) Case with the variation of the shear stress

Fig. 13 Effect of the orientation angle under plane stress condition.

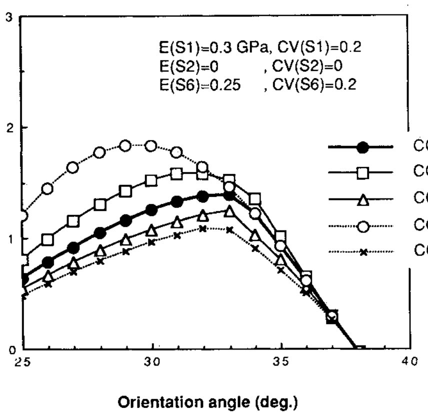


Fig. 14. Effect of the correlation between the longitudinal tensile and shear strengths.

Effect of Correlations Among Strengths and Among Applied Stresses

The principal strengths and the applied stresses are assumed to have no correlations with each other. For the strengths, there may be some correlation with each other since the strength in each direction depends on the strength of the interface between fibers and a matrix to some extent. However, it is very difficult to investigate it experimentally, and the correlation coefficients for the principal strengths of composite materials have not yet been reported.

Many computational results were obtained to clarify the effect of the correlation coefficient between the longitudinal tensile and shear strengths under the plane stress condition. As a result, it is found that the correlation among the strengths does not affect the reliability so much under many stress conditions.

The effect of the coefficient of correlation between the applied stresses is shown in Fig. 14. It is found that the large positive correlation yields higher reliability, whereas the larger negative correlation yields lower reliability. This is because the direction of the resultant applied stress approaches constant as the correlation increases. It should be noted that the maximum reliability orientation angle changes with changing the correlation.

From this result, the maximum reliability orientation does not change very much until the correlation becomes very high. Therefore, the correlation coefficient can be chosen to be zero, for practical purposes, unless its definite value is known.

Concluding Remarks

For an optimum design of composite materials, the reliability of the material plays an important role since it has remarkable anisotropy and optimum configurations, such as fiber orientations, and stacking sequences are affected by uncertainty in the applied load. The reliability analysis of unidirectional fibrous composites is performed from this point of view.

The safety margin of the composites is defined based on the Tsai-Wu failure criterion, and the safety index that represents the reliability is obtained by the proposed method, which is one of the advanced first-order second-moment methods. The extended Lagrangian function and the new search method for multimodal limit state functions are introduced and the method is found to be valid in estimating the reliability of unidirectional composites.

The numerical results clarify various effects on the reliability. The reliability of composites decreases and the fiber orientation angle that gives a maximum reliability changes with the increase in the variation of the applied stress under plane stress conditions. The most important point is that the orientation of maximum reliability changes with increasing the coefficient of variation of the applied stress. This is because the limit state function of fibrous composites is nonlinear, and only the proposed approach can reveal the reliability of composites.

The FOSM method gives an overestimated value of the safety index and sometimes gives a different orientation of maximum reliability. Therefore, the AFOSM method should be used. The type of strength distribution affects the reliability. The Weibull distribution gives a smaller safety index, whereas the log-normal distribution gives a larger safety index. But the differences becomes small when the variance of stress increases.

The variation of the orientation angles also decreases the reliability, but it does not affect the maximum reliability of composites subject to shear. The effect of the correlations among the fundamental strengths and among the applied stress are discussed.

Acknowledgment

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